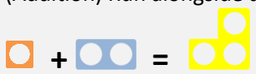
Strand	Representations and Symbols	Written & Mental Methods for Calculations	Order of Calculation	Rounding	Problem Solving
Foundation Stage		*using quantities and objects, add and subtract two single-digit numbers and count on or back to find the answer			*solve problems including doubling

<u>Addition and Subtraction</u>

Begin to relate addition to combining two groups of objects

and subtraction to taking objects away from a group. 

(Addition) Run alongside activities using the Numicon shapes to build conceptual understanding.



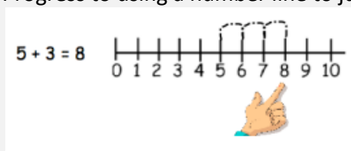
Make a record in pictures, words or symbols of calculation activities carried out.
Model using number sentences alongside practical activities.

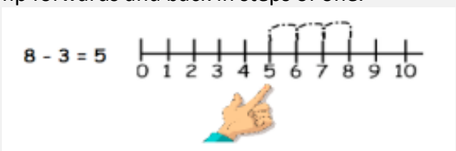
5 + 1 = 6 7 – 3 = 4

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

10 = 5 + 5 3 = 3

Progress to using a number line to jump forwards and back in steps of one.





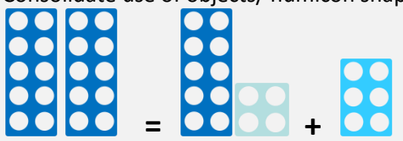
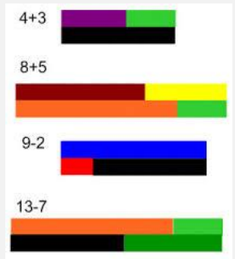
Use games, songs and practical activities to begin using vocabulary.

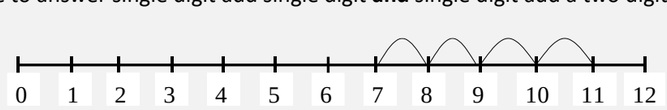
<u>Mental strategies</u>

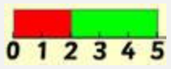
Number doubles of single digits
One more and one less than a given number up to 20

<u>Resources</u>

Numbers and Patterns
Numicon (Firm Foundations Kit)

<table>
<tr>
<td rowspan="2">Year One</td>
<td>*read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs</td>
<td>*represent and use number bonds and related subtraction facts within 20
*add and subtract one-digit and two-digit numbers to 20, including zero</td>
<td></td>
<td></td>
<td>*solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ – 9</td>
</tr>
<tr>
<td colspan="5">
<u>Addition and Subtraction</u>
Consolidate use of objects/ numicon shapes/ cuisenaire rods when calculating.

20 = 14 + 6


Use a number line to answer single digit add single digit and single digit add a two digit number by counting along in ones.

7 + 4 = 11

Cuisenaire rods can be placed along a number line  to calculate addition and subtraction problems.

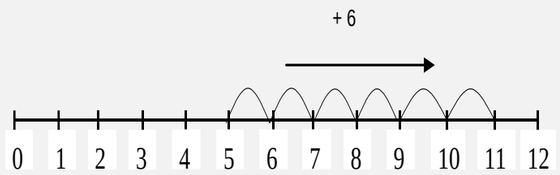
Missing numbers need to be placed in all positions possible.
3+ 4 = □ □ = 3 + 4 7 – 3 = □ □ = 7 – 3
3 + □ = 7 7 = □ + 4 7 - □ = 4 4 = □ - 3
□ + 4 = 7 7 = 3 + □ □ - 3 = 4 4 = 7 – □
□ + ∇ = 7 7 = □ + ∇ □ - Δ = 4 4 = □ - Δ
<u>Mental strategies</u>
Recall and use of addition and subtraction facts up to 10 and then derive to 20.
</td>
</tr>
</table>

Year Two		*recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 *add and subtract numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers	*show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot *recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems		*solve problems with addition and subtraction: • using concrete objects and pictorial representations, including those involving numbers, quantities and measures • applying their increasing knowledge of mental and written methods

To build on conceptual understanding use concrete/ visual resources such as real objects, Cuisenaire rods, numicon, base 10 and bar models.

Developing mental methods through the use of a number line and hundred square to add and subtract two-digit numbers and ones or tens.

Find a 'difference' by counting up;
I have saved 5p. The socks that I want to buy cost 11p. How much more do I need in order to buy the socks?



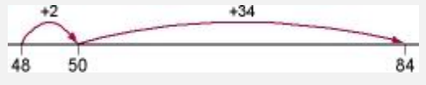
11 – 5 = 6

Developing mental methods for adding two two digit numbers through:

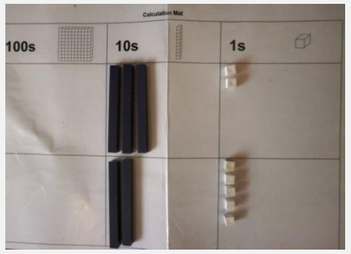
An **empty number line** helps to record the steps to calculating a total. Here are two possible ways to do 48 + 36. There are others.

48 + 36 = 84

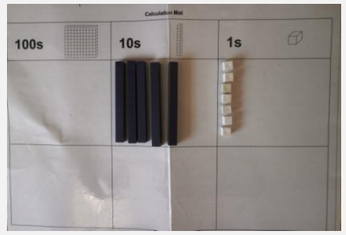
or:



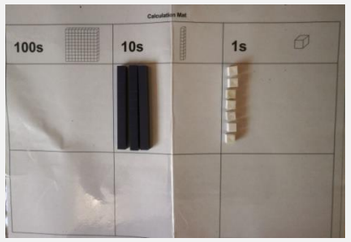
Moving onto using the calculation mat and base 10 to solve adding/ subtracting two two-digit numbers without regrouping.



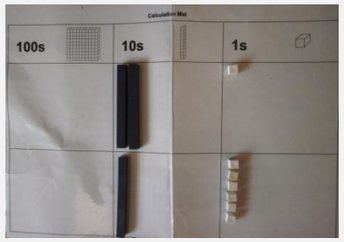
32 +
25
—



32 +
25
57



37-
16
—



37-
16
21

Use jottings and mental strategies, such as bonds, doubles, adjusting to solve adding three one-digit numbers; pairing two numbers and manipulating the third. Knowing that numbers can be added in any order, because of the laws of commutativity:

Any operation * which has the property that a * b = b * a for all members a and b of a given set is called **commutative**.

For the set of real numbers:

Addition is commutative, e.g. $2 + 3 = 3 + 2$.

Multiplication is commutative, e.g. $2 \times 3 = 3 \times 2$.

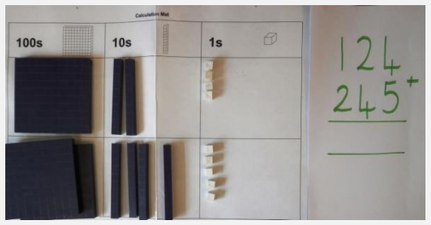
Subtraction and division are not commutative because, as counter-examples, $2 - 3 \neq 3 - 2$ and $2 \div 3 \neq 3 \div 2$.

Mental Strategies

Recall of addition and subtraction facts to 20 and derive facts to 100.
Able to use the inverse calculation to check and calculation.

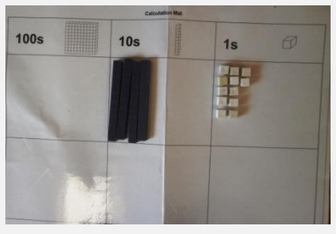
<table>
<tr>
<td rowspan="2">Year Three</td>
<td></td>
<td>*add and subtract numbers mentally, including:
• a three-digit number and ones
• a three-digit number and tens
• a three-digit number and hundreds
*add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
*solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction</td>
<td>*estimate the answer to a calculation and use inverse operations to check answers
*solve problems, including missing number problems, using number facts, place
value, and more complex addition and subtraction</td>
<td></td>
<td>*solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.</td>
</tr>
<tr>
<td colspan="5">Teaching concrete to pictorial to abstract notation by using chips or base 10 grids alongside modelled calculation:

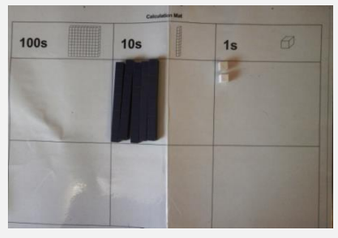
Initially without regrouping:

By modelling regrouping using the base 10 equipment:

47+
5
52

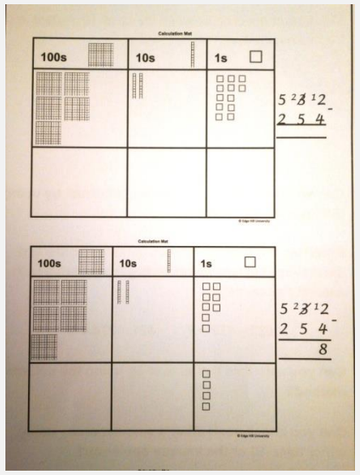
Moving to formal written methods of columnar addition and subtraction:

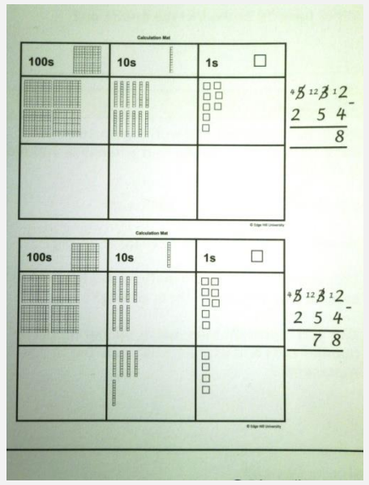
367 + 185 = 431
either or

367 300 + 60 + 7
+185 100 + 80 + 5
12 400 +140+12 = 552
140
400
552</td>
</tr>
</table>

leading to

$$\begin{array}{r} 367 \\ +185 \\ \hline 552 \\ \scriptstyle 1\,1 \end{array}$$





<u>Mental Strategies</u>

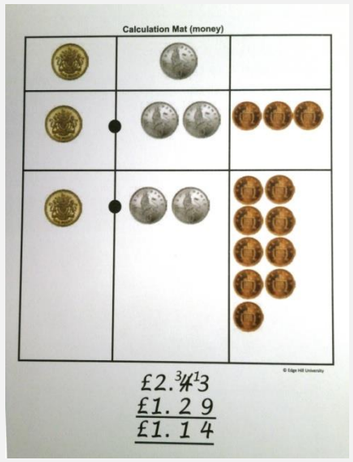
Strategies for adding/ subtracting 9/11 by adding/ subtracting 10 and adjusting by 1. ...19/21

Build on imagery and use of concrete objects in Year 2.

Increase calculations from two-digit to three-digit.

Year Four		*add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	*estimate and use inverse operations to check answers to a calculation		*solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

As for year three addition and subtraction strategies but with up to four digits and moving into the context of money.



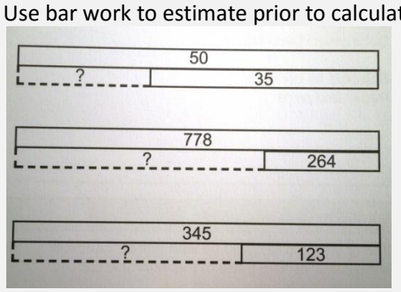
3587 + 675 = 4262

```
  3587
+ 675
 4262
 111
```

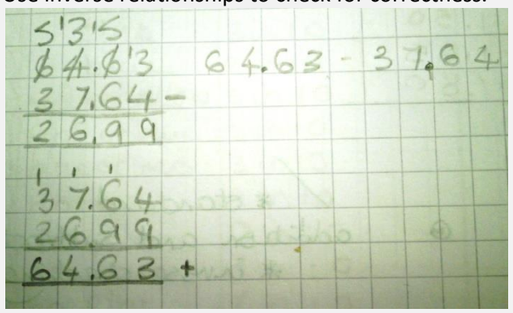
Revert to expanded methods if the children experience any difficulty.

Mental Strategies

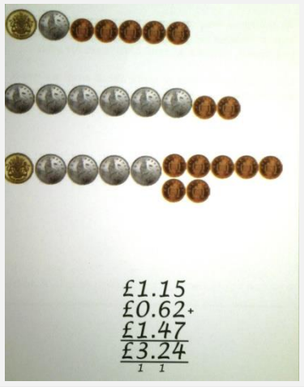
Use bar work to estimate prior to calculation.



Bridging through the next multiple of 10, 100 or 1000.
Identifying near doubles.

<table>
<tr><td></td><td colspan="5">Add or subtract the nearest multiple of 10, 100, 1000 and adjust.
Use inverse relationships to check for correctness.
 
</td></tr>
<tr><td rowspan="2">Year Five</td><td></td><td>*add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
*add and subtract numbers mentally with increasingly large numbers</td><td></td><td>*use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy</td><td>*solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</td></tr>
<tr><td colspan="5">Any operation ● which has the property that a ● (b ● c) = (a ● b) ● c for all members a, b and c of a given set is called associative.
For the set of real numbers:
Addition is associative, e.g. 1 + (2 + 3) = (1 + 2) + 3.
Multiplication is associative, e.g. 1 × (2 × 3) = (1 × 2) × 3.
Subtraction and division are not associative because, as counter examples, 1 – (2 – 3) ≠ (1 – 2) – 3 and 1 ÷ (2 ÷ 3) ≠ (1 ÷ 2) ÷ 3.
We can use the associative law to help with multiplication calculations. For example:
Find 5 × 26.
Factorise 26 as 13 × 2, so we now have 13 × 2 × 5. Use the associative law to associate the 2 with the five, rather than with the 13 in order to make the calculation easier. (13 × 2) × 5 = 13 × (2 × 5) = 13 × 10 = 130.

Use a number line for small differences e.g. 4007 – 3982 and negative numbers.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).
72.8
+54.6
127.4
1 1
 
</td></tr>
</table>

<table>
<tr>
<td rowspan="2">Year Six</td>
<td></td>
<td></td>
<td>*use their knowledge of the order of operations to carry out calculations involving the four operations</td>
<td></td>
<td>*solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
*solve problems involving addition, subtraction, multiplication and division
*use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.</td>
</tr>
<tr>
<td colspan="5">Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.
13.86 + 9.481 = 23.341

13.86
+ 9.481
23.341
1 1 1

Continued use of inverse and estimation strategies for checking including rounding for decimals.

To use a systematic approach when solving calculations
e.g. 36 - □ = 5 x 6
36 - □ = 30

To be able to determine which is the most efficient method to use and manipulate the numbers to best suit the method.

When there are no brackets in an expression, do multiplication or division before addition or subtraction, e.g. 4 + 3 × 7 = 4 + 21 = 25.

When there are brackets in an expression, do the operation inside the brackets first, e.g. (4 + 3) × 7 = 7 × 7 = 49.

These rules are called the order of operations.

Additional User Example

An old method of remembering this is by using the BODMAS rule:

B = Brackets first
O = Order or powers (Order means anything raised to the power of a number)
D = Division
M = Multiplication
A = Addition
S = Subtraction

Your scientific calculator will default to this order unless you put in brackets to get it to do a different order.

An arithmetic calculator will not do the operations in the correct order, unless you put the operations into the arithmetic calculator in the correct order and press equals inbetween each operation</td>
</tr>
</table>

	Ext: (L6) One operation ? is **distributive** over another operation * if (*a* * *b*) ? *c* = (*a* ? *c*) * (*b* ? *c*) for all members *a*, *b* and *c* of a given set. For the set of real numbers: Multiplication is distributive over addition and subtraction, e.g. (50 + 6) × 4 = (50 × 4) + (6 × 4) and (30 – 2) × 4 = (30 × 4) – (2 × 4). Division is distributive over addition and subtraction, e.g. (40 + 8) ÷ 4 = (40 ÷ 4) + (8 ÷ 4) and (60 – 4) ÷ 4 = (60 ÷ 4) – (4 ÷ 4). Addition and subtraction are not distributive over other number operations. We can use the distributive law to help with multiplication calculations, for example 5 × 26. Partition 26 as 20 and six separately, then multiply the twenty and the six separately by 5, to get 100 and 30, which add up to 130. Thus the multiplication by 5 is being distributed across addition of 20 and 6. 5 × (20 + 6) = (5 × 20) + (5 × 6) We can also use the distributive law to help us with division, for example 96 ÷ 6. Partition 96 as 60 and 36, then divide the 60 and the 36 by six separately To get ten and six, which add up to 16. Thus the division by 6 is being distributed across the addition of 60 and 36. (60 + 36) ÷ 6 = (60 ÷ 6) + (36 ÷ 6) The distributive law involves partitioning numbers into those that can be more easily calculated e.g. 56 × 4 = (50 + 6) × 4 = (50 × 4) + (6 × 4) = 200 + 24 = 22 46 × 98 = 46 × (100 – 2) = (46 × 100) – (64 × 2) = 4600 – 92 = 4508 48 ÷ 4 = (40 + 8) ÷ 4 = (40 ÷ 4) + (8 ÷ 4) = 10 + 2 = 12 The understanding of how to apply these laws with numbers lays the foundations for success with algebra later on.